5.

1. For a pair (w_i, x_i),

$$p(w_i, x_i | \lambda, \pi) = \left[\frac{5}{7} \frac{\lambda^{x_i} e^{-\lambda}}{x_i!}\right]^{w_i} \left[\frac{2}{7} \pi^{x_i} (1-\pi)^{1-x_i}\right]^{1-w_i}$$

$$\therefore \log p(W, X | \lambda, \pi) = \sum_{i=1}^{N} w_i \left[\log \frac{5}{7} + x_i \log \lambda - \lambda - \log x_i!\right] + (1-w_i)\left[\log \frac{2}{7} + x_i \log \pi + (1-x_i) \log(1-\pi)\right]$$

2) $$F(q, \pi, \lambda) = E_q\left[\log p(W, X | \lambda, \pi)\right] + H(q)$$

$$= \sum_{i=1}^{N} q_i \left[\log \frac{5}{7} + x_i \log \lambda - \lambda - \log x_i!\right] + (1-q_i)\left[\log \frac{2}{7} + x_i \log \pi + (1-x_i)\log(1-\pi)\right] + H(q)$$

3) a $q_i := q_i(w_i = 1) = p(w_i = 1 | x_i, \lambda, \pi) \propto \quad \ldots$

(You can do)

b) $$\frac{\partial F}{\partial \lambda} = 0 \Rightarrow \sum_{i=1}^{N} q_i \left[\frac{x_i}{\lambda} - 1\right] = 0$$

$$\Rightarrow \lambda = \sum_{i=1}^{N} q_i x_i \Big/ \sum_{i=1}^{N} q_i$$

Similarly, $\pi = \frac{1}{N} \sum_{i=1}^{N} q_i$